

Riemann boundary value problem on nonrectifiable curves for certain Beltrami equations

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Abstract

Copyright © 2018 John Wiley & Sons, Ltd. In this paper, we obtain a Cauchy-type integral representation to solve certain Beltrami equations and apply it for research of the Riemann boundary value problem for that equations on nonrectifiable contours. While more or less studied for piecewise smooth contours, for the case of nonrectifiable curve, this is a pioneer result. There is only one relatively simple type of Beltrami equation that is solved under such condition (see below). Our next goal is to achieve solutions for more types of the Beltrami equations on nonrectifiable curves.

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Keywords

Beltrami equation, Marcinkiewicz exponent, nonrectifiable curve

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